MECHANICS

CH.1 - DYNAMIC

Rectilinear Motion

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KING OF THE VECTORS

Ch(1) Rectilinear Motion

① Motion with uniform acceleration

حركة بعجلة ثابتة او منتظمة

a = Constant or uniform

② Motion with variable acceleration

حركة بعجلة متغيرة

a = f(variable)

① Motion with uniform acceleration

* اذا كان الجسم يتحرك في خط مستقيم بعجلة ثابتة او منتظمة يتم تطبيق قوانين نيوتن للحركة

$$① \quad V = V_0 \pm at$$
$$② \quad V^2 = V_0^2 \pm 2a(S - S_0)$$
$$③ \quad (S - S_0) = V_0 t \pm \frac{1}{2} a t^2$$

* where:

V:- Final velocity السرعة النهائية

V_0:- initial velocity السرعة الابتدائية

S:- Final displacement الازاحة النهائية

S_0:- initial displacement الازاحة الابتدائية

t:- time الزمن

d:- Total distance المسافة الكلية

a:- Linear acceleration العجلة الخطية

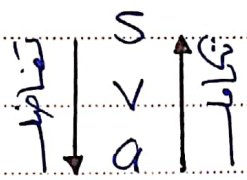
g:- gravity = 9,81 m/s^2 عجلة الجاذبية

[Linear acceleration]

- acceleration (a)
  - acceleration (+a)
  - deceleration (-a)
- gravity (g)
  - downward (+9,81)
  - upward (-9,81)

② Motion with Variable acceleration

* إذا كان الجسم يتحرك في خط مستقيم بعجلة متغيرة فيتم تطبيق القانون المناسب حسب تغير نوع العجلة

$a=f(t)$	$a=f(s)$	$a=f(v)$	$v=f(s)$
	$\int a\,ds = \int v\,dv$ (اثبات) $a = \frac{dv}{dt} * \frac{ds}{ds}$	$a = \frac{dv}{dt}$	لو معطى زمن في السؤال $v = \frac{ds}{dt}$ لو معطى ازاحة في السؤال $\int a\,ds = \int v\,dv$
$v = \frac{ds}{dt} = s^{\cdot}$	$a = \frac{dv}{ds} * \frac{ds}{dt}$		
$a = \frac{dv}{dt} = v^{\cdot} = s^{\cdot\cdot}$	$a = \frac{dv}{ds} * v$		
$(v - v_0) = \int a\,dt$	$a\,ds = v\,dv$		
$(s - s_0) = \int v\,dt$			

* ملاحظات هامة جداً

① لو قال انه الجسم بدأ حركته من السكون ← $(v_0 = 0 \leq t_0 = 0)$ ← (from rest)

② لو قال انه الجسم بدأ حركته من نقطة الاصل ← $(s_0 = 0 \leq t_0 = 0)$ ← (from origin)

③ لو قال انه الجسم اتحرك في اتجاه اليمين ← $(v = + \leq s = +)$ ← (To Right)

④ لو قال انه الجسم غير اتجاه حركته ← $(v = 0)$ ← (change it's direction)

⑤ لو قال انه الجسم وصل لاقصى ازاحة ← $(v = 0)$ ← (Max displacement)

⑥ لو لم يعطى اى معلومة عن الازاحة الابتدائية والسرعة الابتدائية اعتبرهم بصفر ← $(s_0 = v_0 = 0)$

⑦ لو قال انه العجلة بصفر تبقى السرعة ثابتة ← $(a = 0)$ ← $(v = \text{Const})$ ← $v = \frac{s}{t}$

EX:1 :- Traveling with an initial speed of (70 km/h) a car accelerated at (6000 km/h²) along a straight road. How long will it take to reach a speed of (120 km/h)? Also through what displacement does the car travel during this time?

Data * Sol * req

$V_o = 70$ km/h — $t = ?$

$a = 6000$ km/h² — $S = ?$

$V = 120$ km/h

$\because a = \text{constant} = 6000$ km/h²

$\therefore V = V_o + at$

$120 = 70 + (6000)t$

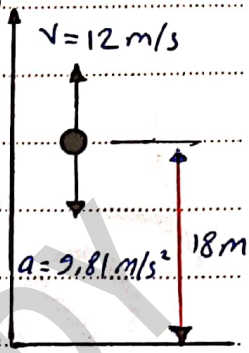
$\therefore t = \frac{1}{120} = 8.3 * 10^{-3}$ h $= 30$ sec #

$S - S_o = V_o t + \frac{1}{2} a t^2$

$S = (70)(\frac{1}{120}) + \frac{1}{2}(6000)(\frac{1}{120})^2$

$\therefore S = 0.79$ km #

EX: 2 :- A Ball is Thrown from the Top of atwer (18m) y height, with a velocity of (12 m/s) vertically upward knowing that the acceleration of the ball is constant and equal to ($9,81\ m/s^2$) downward. Determine (a) the velocity & elevation (y) of the ball above the ground at any time (t). (b) the heighest elevation and the corresponding value of (t). (c) the time when the ball will hit the ground and the corresponding velocity ?



* Sol *

Data :- $g = -9,81\ m/s^2 \uparrow$, $y_0 = 18\ m$, $V_0 = 12\ m/s$

(a) $V = V_0 + gt$

$V = 12 - 9,81t \longrightarrow ①$ # (a)

$y - y_0 = V_0 t + \frac{1}{2} g t^2$

$y = 18 + 12t - 0,5(9,81)t^2 \longrightarrow ②$ # (a)

(b) max height = heighest elevation $\longrightarrow$ at $V = 0$ For eq ①

$0 = 12 - 9,81t$

$\therefore t = 1,22$ sec # (b) $\longrightarrow$ at $t = 1,22$ For eq ②

$y = 18 + 12(1,22) - 0,5(9,81)(1,22)^2$

$\therefore y_{max} = 25,33\ m$ # (b)

(c) The ball hit the ground $\longrightarrow$ at $y = 0$ For eq ②

$-0,5(9,81)t^2 + 12t + 18 = 0 \longrightarrow$ (حل معادلة تربيعية)

$t = 3,49$ sec # (c) , $t = -1,05$ sec (مرفوض)

at $t = 3,49$ sec $\longrightarrow$ for eq ①

$V = 12 - 9,81(3,49)$

$\therefore V = -22,2\ m/s \downarrow$ # (c)

EX: 3 :- A sphere is fired down ward into a medium with an initial speed of (27 m/s) if it a deceleration $a = (-6t)\ m/s^2$. where (t) is in seconds. determine the displacement Traveled when it Stoped ?

Data:-

$V_0 = 27\ m/s$

$a = -6t = a\,f(t)$ [variable]

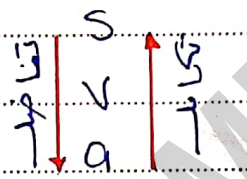
When it Stoped $\rightarrow V = 0$

$a = -6t$

Sol

Req:-

$S = ?$



$$V - V_0 = \int_0^t a\,dt$$

$$V - 27 = \int_0^t -6t\,dt$$

$$V - 27 = \frac{-6}{2} t^2 \Big|_0^t$$

$$V - 27 = -3t^2$$

$$V = -3t^2 + 27$$

when it Stoped $\rightarrow$ at $V = 0$

$$0 = -3t^2 + 27$$

$$\therefore t = 3\ sec$$

$$S - S_0 = \int_0^t V\,dt$$

$$S - 0 = \int_0^t -3t^2 + 27\,dt$$

$$S = \frac{-3}{3} t^3 + 27t \Big|_0^t$$

$$S = -t^3 + 27t$$

at $t = 3\ sec$

$$S = -(3)^3 + 27(3)$$

$$\therefore S = 54\ m$$ #

EX:4 :- The acceleration of particle traveling along a straight line is $(a = 0.25 s^{\frac{1}{2}})\ m/s^2$. where (s) is in meters if $(v=0) \leq (s=1\ m)$ when $(t=0)$. Determine the particles velocity at $(s=2)\ m$

Data:

$a = 0.25\ s^{\frac{1}{2}}$ $= a = f(s)$ [variable]

$v_0 = 0$ $t_0 = 0$

$s_0 = 1\ m$ $s = 2\ m$

* Sol *

Req.

$v = ?$

$$\int_{s_0}^{s} a\,ds = \int_{v_0}^{v} v\,dv$$

$$\int_{1}^{s} 0.25\, s^{\frac{1}{2}}\,ds = \int_{0}^{v} v\,dv$$

$$\frac{0.25}{1.5}\, s^{\frac{3}{2}} \Big|_1^s = \frac{v^2}{2} \Big|_0^v$$

$$0.16\,(s^{\frac{3}{2}} - 1) = \frac{v^2}{2} \qquad * 2$$

$$v^2 = 0.32\,(s^{\frac{3}{2}} - 1)$$

$$v = \left[0.32\,(s^{\frac{3}{2}} - 1)\right]^{\frac{1}{2}}$$

at $s = 2\ m$

$$\therefore v = \left[0.32\,(2^{\frac{3}{2}} - 1)\right]^{\frac{1}{2}} = 0.76\ m/s \quad \#$$

EX: 5 :- A small particle is fired vertically downward into a fluid medium with an initial velocity of (60 m/s). due To the drag resistance of the fluid the particle experiences a deceleration ($a = -0.4V^3$) m/s^2. Determine the particle's velocity and position after (4s) it is fired.

Data:- * Sol * Req:-

$V_0 = 60$ m/s

$a = -0.4V^3$ m/s^2 = $a = f(V)$ ⟶ [variable]

$t = 4$ sec

Req:- $V = ?$, $S = ?$

$$a = \frac{dV}{dt}$$

$$-0.4V^3 = \frac{dV}{dt}$$

$$\int_{t_0}^{t} -0.4\,dt = \int_{V_0}^{V} \frac{dV}{V^3}$$

$$\int_{0}^{t} -0.4\,dt = \int_{60}^{V} V^{-3}\,dV$$

$$-0.4t\Big|_0^t = \frac{V^{-2}}{-2}\Big|_{60}^{V} \quad |*-2$$

$$0.8t = \left(\frac{1}{V^2} - \frac{1}{60^2}\right)$$

at $t = 4$ sec

$\therefore V = 0.55$ m/s #

$$\int a\,ds = \int V\,dV$$

$$\int_{S_0}^{S} -0.4V^3\,ds = \int_{V_0}^{V} V\,dV$$

$$\int_{0}^{S} -0.4\,ds = \int_{60}^{V} \frac{1}{V^2}\,dV$$

$$\int_{0}^{S} -0.4\,ds = \int_{60}^{V} V^{-2}\,dV$$

$$-0.4S\Big|_0^S = \frac{V^{-1}}{-1}\Big|_{60}^{V} \quad |*-1$$

$$0.4S = \left(\frac{1}{V} - \frac{1}{60}\right)$$

at $V = 0.55$

$\therefore S = 4.5$ m #

EX:6 :- The acceleration of particle moves in a straight line is $(a = 5 - 2v)\ m/s^2$ where (v) is it's velocity if the particle begins it's motion from rest. Find the time needed to the velocity becomes (1.25) m/s . * Sol *

Data:- Req.:-

$a = 5 - 2v\ m/s^2 \longrightarrow a = f(v)$ [variable] $t = ?$

$v_o = 0 \quad < \quad v = 1.25\ m/s$

$$a = \frac{dv}{dt}$$

$$5 - 2v = \frac{dv}{dt}$$

$$\int_{t_o}^{t} dt = \int_{v_o}^{v} \frac{dv}{5 - 2v}$$

$$\int_{o}^{t} dt = \frac{1}{-2} \int_{o}^{v} \frac{-2}{5 - 2v}\, dv$$

$$t \Big|_{o}^{t} = -\frac{1}{2} Ln(5 - 2v) \Big|_{o}^{v}$$

$$t = -\frac{1}{2} \left[Ln(5 - 2v) - Ln(5) \right]$$

$$t = -\frac{1}{2} Ln\left(\frac{5 - 2v}{5}\right)$$

at $v = 1.25 \quad < \quad \therefore t = 0.34$ sec #

Ex:7 :- A particle moves in a straight line such that $(v = \frac{2}{1+s})$ cm/s where (v) is the velocity, Find the time needed for particle to arrive the point (s = 4 cm). Find also it's acceleration, if the particle begins it's motion from rest.?

Data:- * Sol * Req:-

$v = \frac{2}{1+s} \rightarrow v = f(s)$ $\quad a = ?$

$s = 4$ cm $\quad t = ?$

$v = \frac{ds}{dt}$

$\frac{2}{(1+s)} = \frac{ds}{dt}$ $\quad \times dt$

$\int_{t_0}^{t} 2\,dt = \int_{s_0}^{s} (1+s)\,ds$

$\int_{0}^{t} 2\,dt = \int_{0}^{s} (1+s)\,ds$

$2t\Big|_0^t = s + \frac{s^2}{2}\Big|_0^s$

$2t = s + \frac{s^2}{2}$ $\quad \div 2$

$t = \frac{s}{2} + \frac{s^2}{4}$

at $s = 4$ $\quad \therefore t = 6$ sec #

$a\,ds = v\,dv$ $\quad \div ds$

$a = v\frac{dv}{ds}$

$v = \frac{2}{(1+s)}$

$\frac{dv}{ds} = 2(1+s)^{-1} = -2(1+s)^{-2}$

$\frac{dv}{ds} = \frac{-2}{(1+s)^2}$

$a = \left(\frac{2}{(1+s)}\right)\left(\frac{-2}{(1+s)^2}\right)$

$a = \frac{-4}{(1+s)^3}$

at $s = 4$ $\quad \therefore a = \frac{-4}{125}$ cm/s² #

Ex:8 :- The acceleration of particle if it moves along a straight line is given by $(a = 2t - 9)\ m/s^2$. when (t) is in seconds if $(S = 1\ m)$ & $(V = 10\ m/s)$ at $(t = 0)$. Determine the particle's velocity and position when $(t = 9\ sec)$. Also determine the total distance of the particle's travel during the time period.

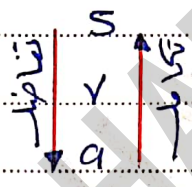
Data:-

$a = 2t - 9$

$t = 0$ & $S_0 = 1\ m$ & $V_0 = 10\ m/s$

$t = 9\ sec$

Sol



Req:-

$V = ?$

$S = ?$

$d = ?$

$a = f(t)$ [variable]

$$V - V_0 = \int_0^t a\,dt$$

$$V - 10 = \int_0^t 2t - 9\,dt$$

$$V - 10 = \frac{2}{2}t^2 - 9t \Big|_0^t$$

$$V = t^2 - 9t + 10 \longrightarrow (1)$$

at $t = 9$

$$\therefore V = (9)^2 - 9(9) + 10$$

$$V = 10\ m/s \ \#$$

$$S - S_0 = \int_0^t V\,dt$$

$$S - 1 = \int_0^t t^2 - 9t + 10\,dt$$

$$S - 1 = \frac{t^3}{3} - \frac{9}{2}t^2 + 10t$$

$$S = \frac{1}{3}t^3 - \frac{9}{2}t^2 + 10t + 1 \longrightarrow (2)$$

at $t = 9$

$$\therefore S = \frac{1}{3}(9)^3 - \frac{9}{2}(9)^2 + 10(9) + 1$$

$$\therefore S_{(9)} = -30.5\ m \ \#$$

* لإيجاد ازمنة التوقف للجسم نضع السرعة تساوي صفر

$V = t^2 - 9t + 10 \longrightarrow$ at $V = 0$

$t^2 - 9t + 10 = 0 \longrightarrow$ حل معادلة تربيعية بالآلة

$\therefore t = 1.29$ sec & $t = 7.70$ sec

* نقوم بالتعويض بازمنة التوقف في معادلة الإزاحة ② لمعرفة اماكن توقف الجسم على المسار.

at $t = 1.29$ sec

$$S = \frac{1}{3}t^3 - \frac{9}{2}t^2 + 10t + 1$$

$$S = \frac{1}{3}(1.29)^3 - \frac{9}{2}(1.29)^2 + 10(1.29) + 1$$

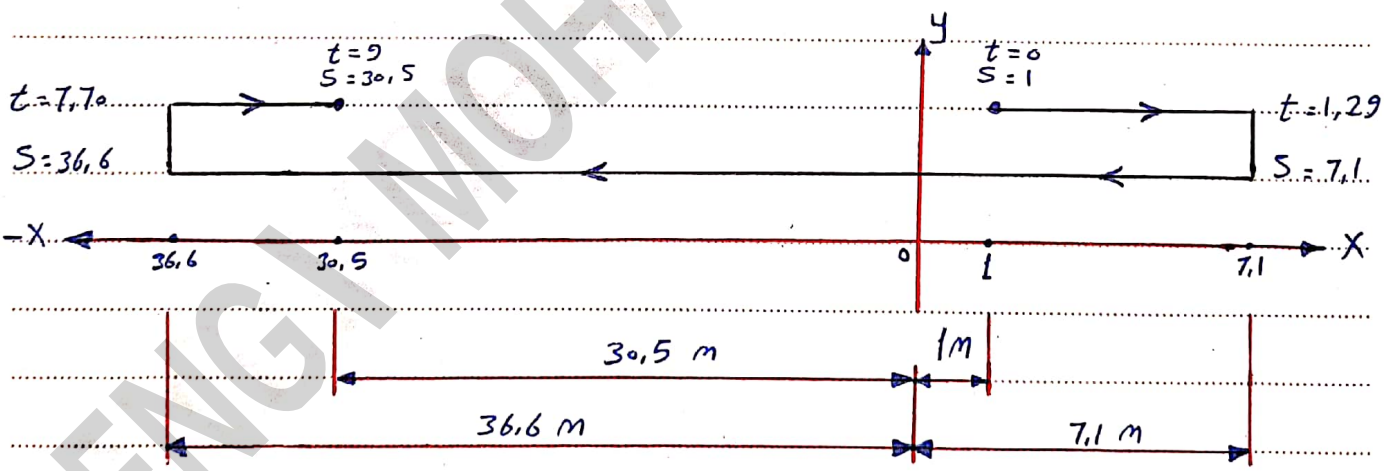
$$S_{(1.29)} = 7.1 \text{ m}$$

at $t = 7.70$ sec

$$S = \frac{1}{3}t^3 - \frac{9}{2}t^2 + 10t + 1$$

$$S = \frac{1}{3}(7.70)^3 - \frac{9}{2}(7.70)^2 + 10(7.70) + 1$$

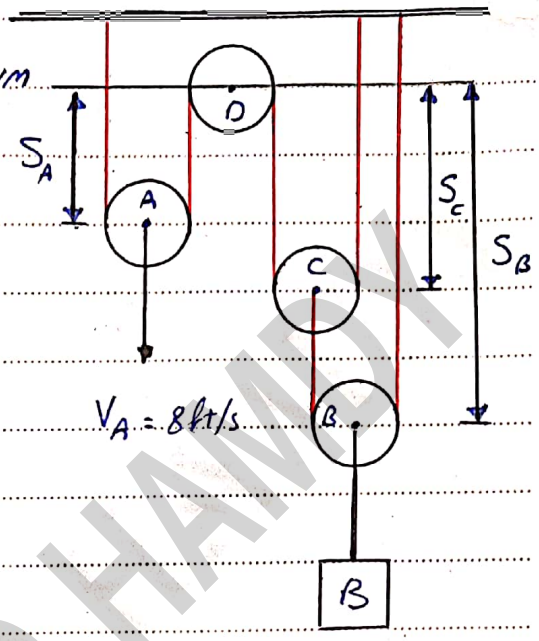
$$S_{(7.70)} = -36.6 \text{ m}$$



Total distance (d) $= (7.1 - 1) + 7.1 + 36.6 + (36.6 - 30.5)$

$d = 55.9 \simeq 56$ m

Ex: 9 :- Determine the velocity of the Block (B) if the cord (A) is pulled down with a speed of (8 ft/s)



* Sol *

datum ا حدد طول كل حبل من ال

c. تفاضل طول كل حبل بالنسبة

1- Total length of first wire :-

$$L_1 = 2S_A + 2S_c \longrightarrow (1)$$

2- Total length of Second wire :-

$$L_2 = (S_B - S_c) + S_B \longrightarrow (2)$$

+ ↑
- ↓

* تفاضل المعادلة (1) ، (2) لايجاد السرعات

$$0 = 2V_A + 2V_c$$

$\because V_A = -8$ ft/s ↓ $\longrightarrow$ $\therefore V_c = 8$ ft/s ↑

$$0 = 2V_B - V_c$$

$\because V_c = 8$ ft/s ↑ $\longrightarrow$ $\therefore V_B = 4$ ft/s ↑ *

MECHANICS

DYNAMIC

ASSIGNMENT 1

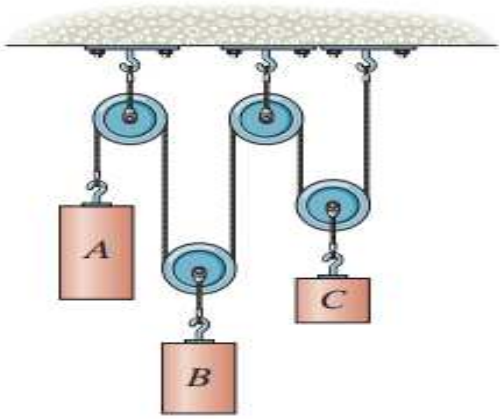
① A particle travels to the right along a straight line with a velocity ($v = \frac{5}{4+s}$) m/s where (s) is in meters. Determine its deceleration when (s = 2 m) ?

② when a train is traveling along a straight track at (2 m/s). it begins to accelerate at ($a = 60\ v^{-4}$) m/s^2 where (v) is in m/s . determine its velocity (v) and the position after (3 sec) ?

③ A particle travels in a straight line with accelerated motion such that (a= - ks) where (s) is displacement from the starting point and (k) is a constant which is to be determined for (s = 2 ft) the velocity is (4 ft/s) and for (s = 3.5 ft) the velocity is (10 ft/s) . what is (s) when (v = 0) ?

④ A particle begins its motion in a straight line such that its position relative to a fixed point on that straight line is given by (S = t3 - 9t2 + 15t + 5). Find the position of the particle when the acceleration vanished, find also the total distance ?

⑤ the pulley system supports three blocks as shown , if (A) is moving downward with a speed of (12 ft/s), while (C) is moving up with speed of (4 ft/s), what is the speed of the block (B) ?



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